A Fresh View at Type Inference

Yin Wang
October 19, 2012

Location: LH 101
Time: 4:15 PM

What is λ-calculus?

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Three kinds of terms of λ-calculus

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Three kinds of terms of λ-calculus

variable	x
function (lambda)	λx.E
function application	E_1E_2

What are they?

variable

x y z

function

λx. E

function application

E1E2

What are they?

wires

x y z

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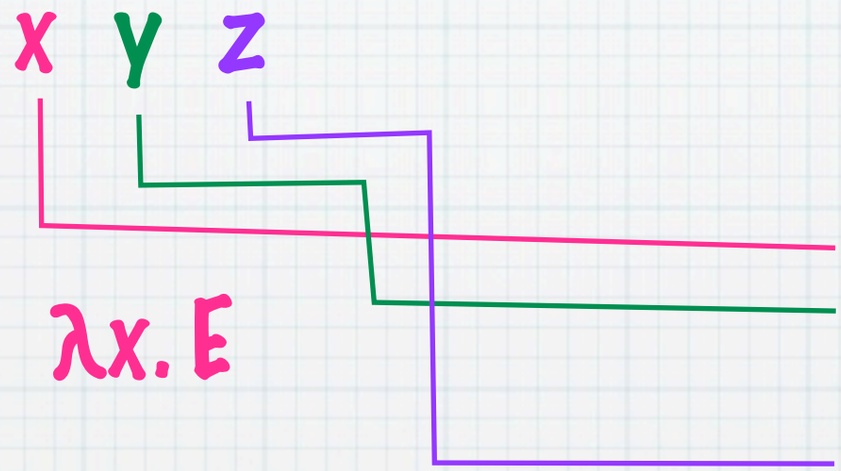
wires



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$E_1 E_2$

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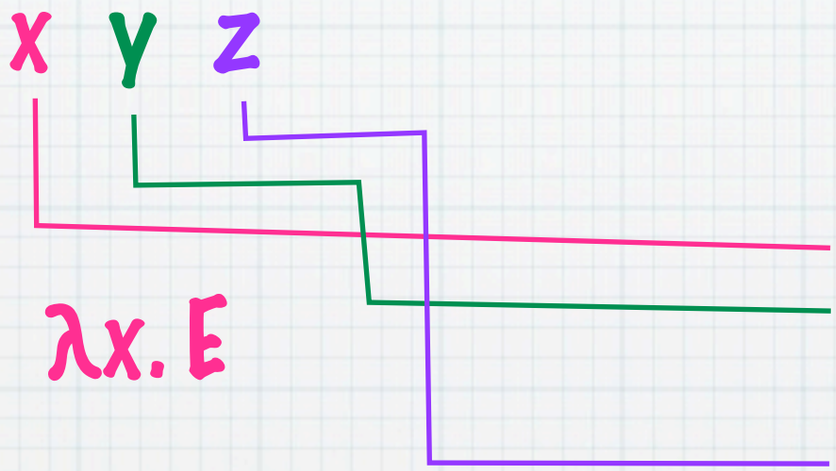

circuit templates

function application

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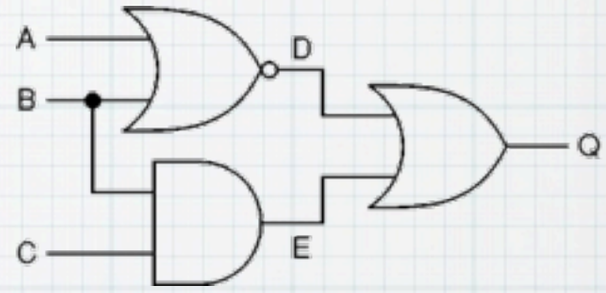

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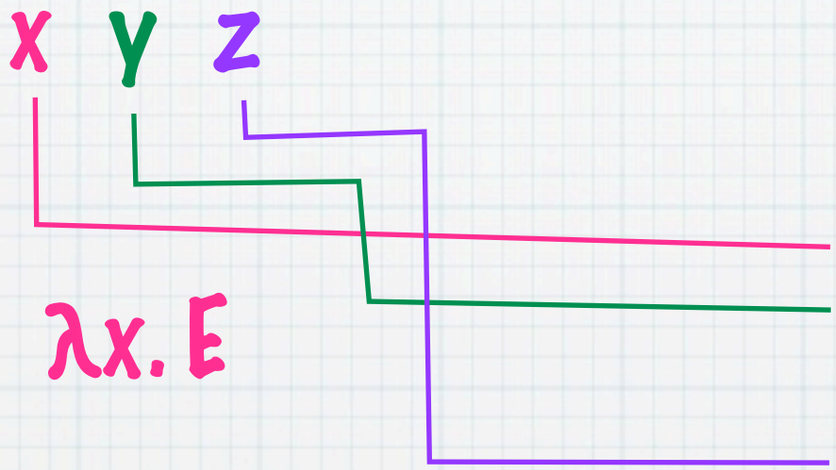

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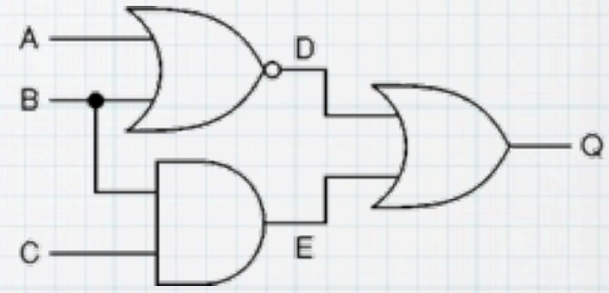

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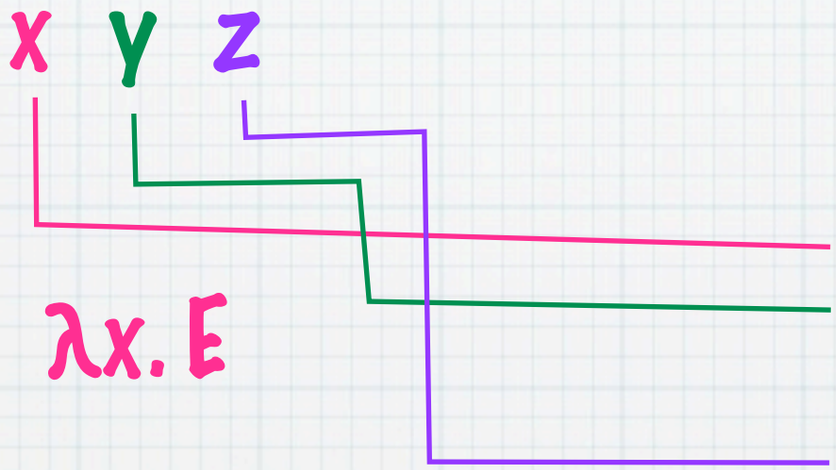

circuit templates

$\lambda x.E$

circuit template instantiation and connection

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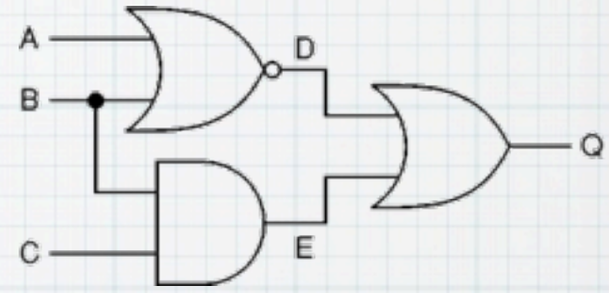

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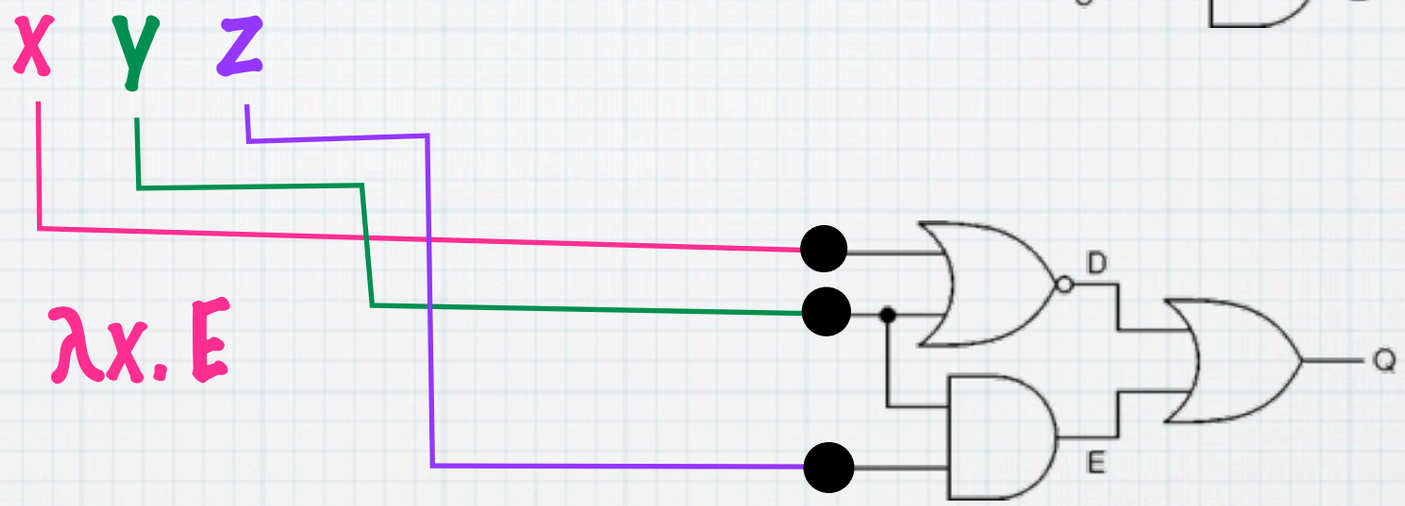

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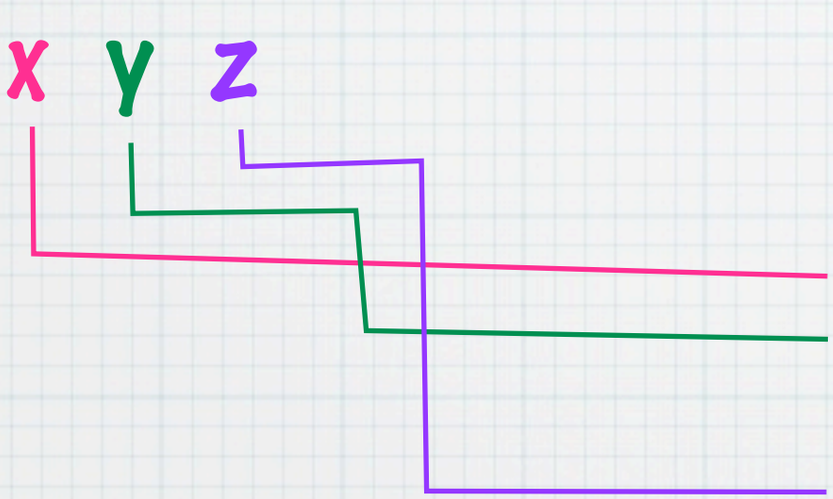

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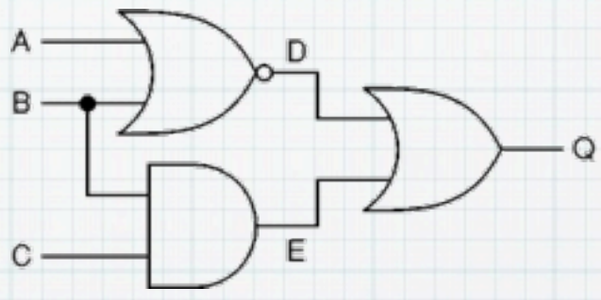

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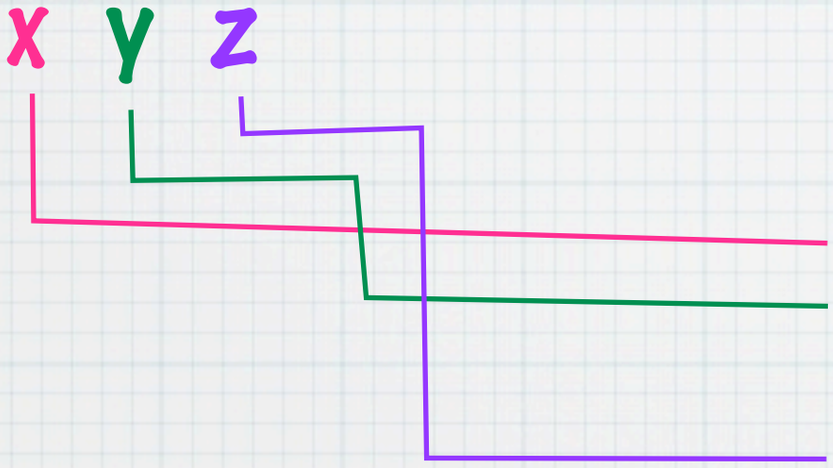

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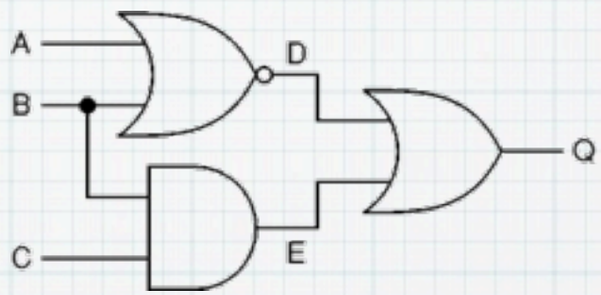

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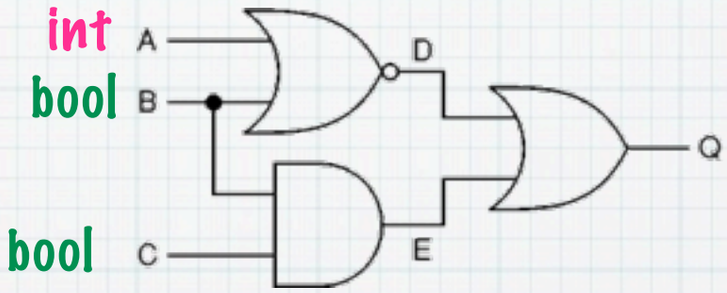

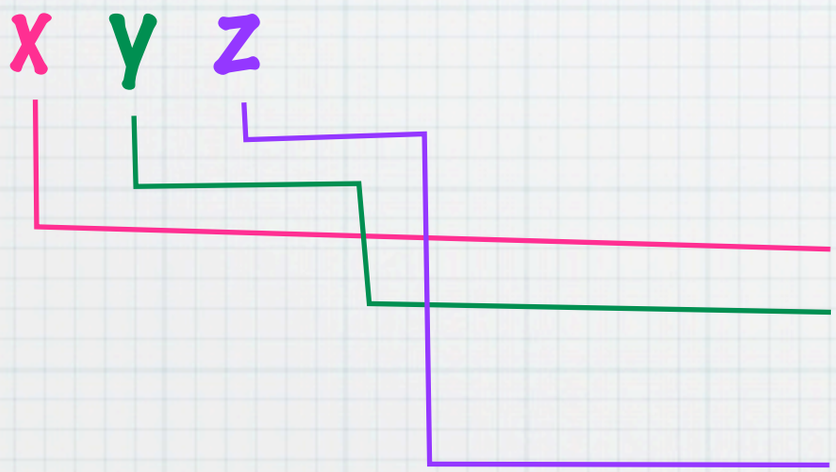

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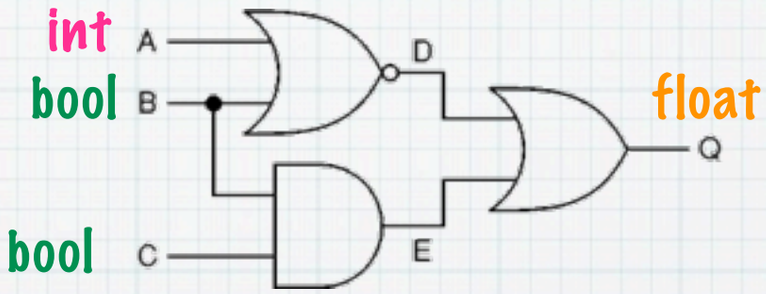

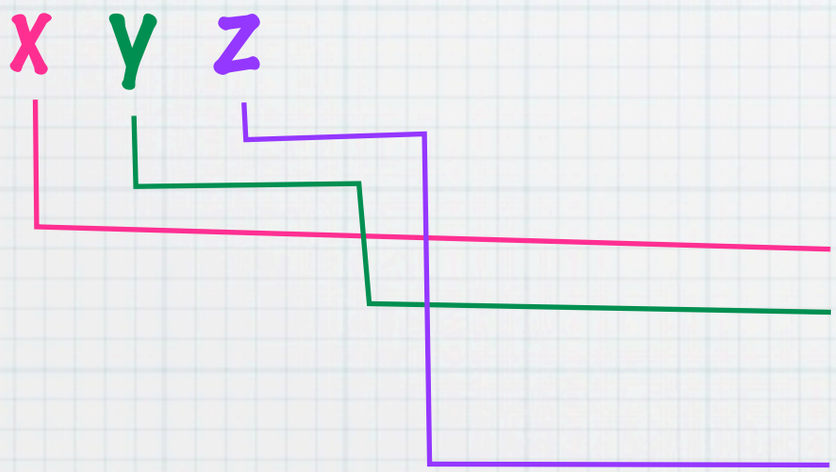

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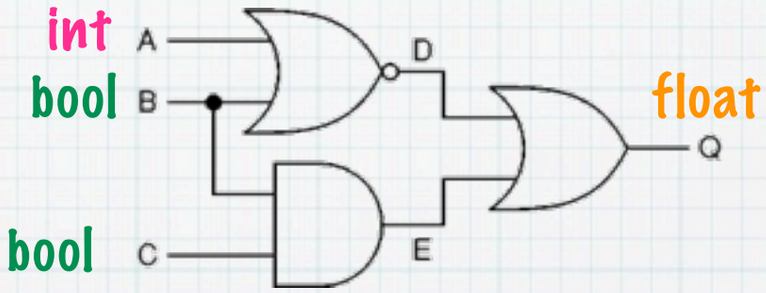

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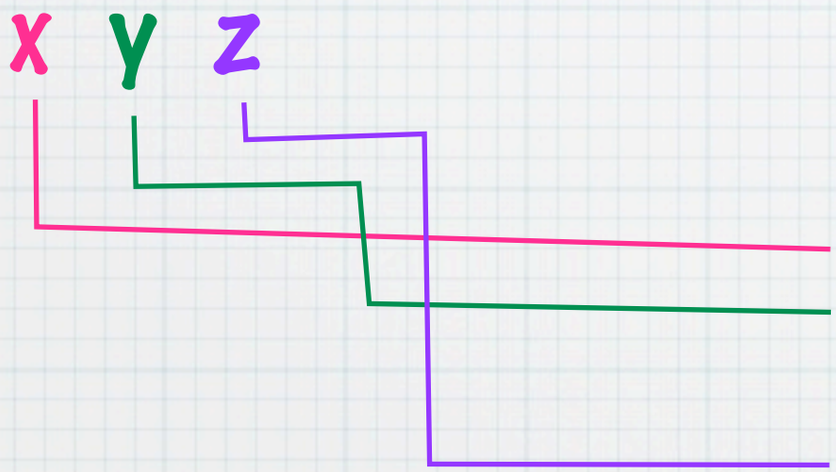
(int, bool, bool) → float



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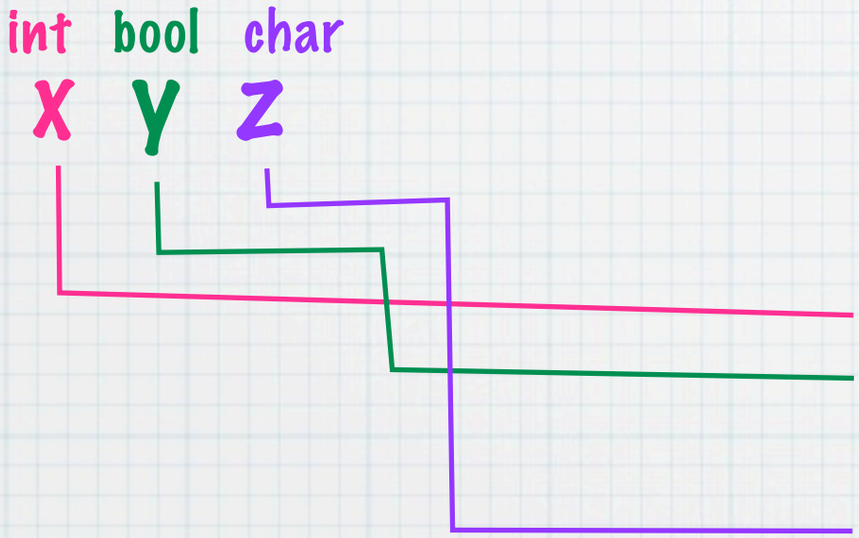

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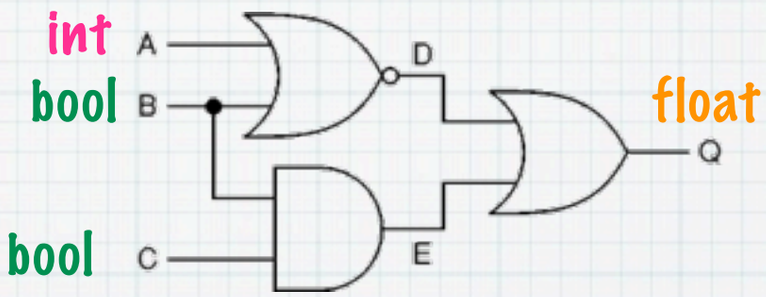

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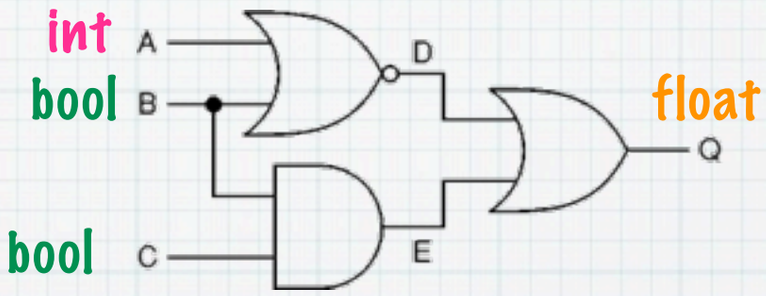

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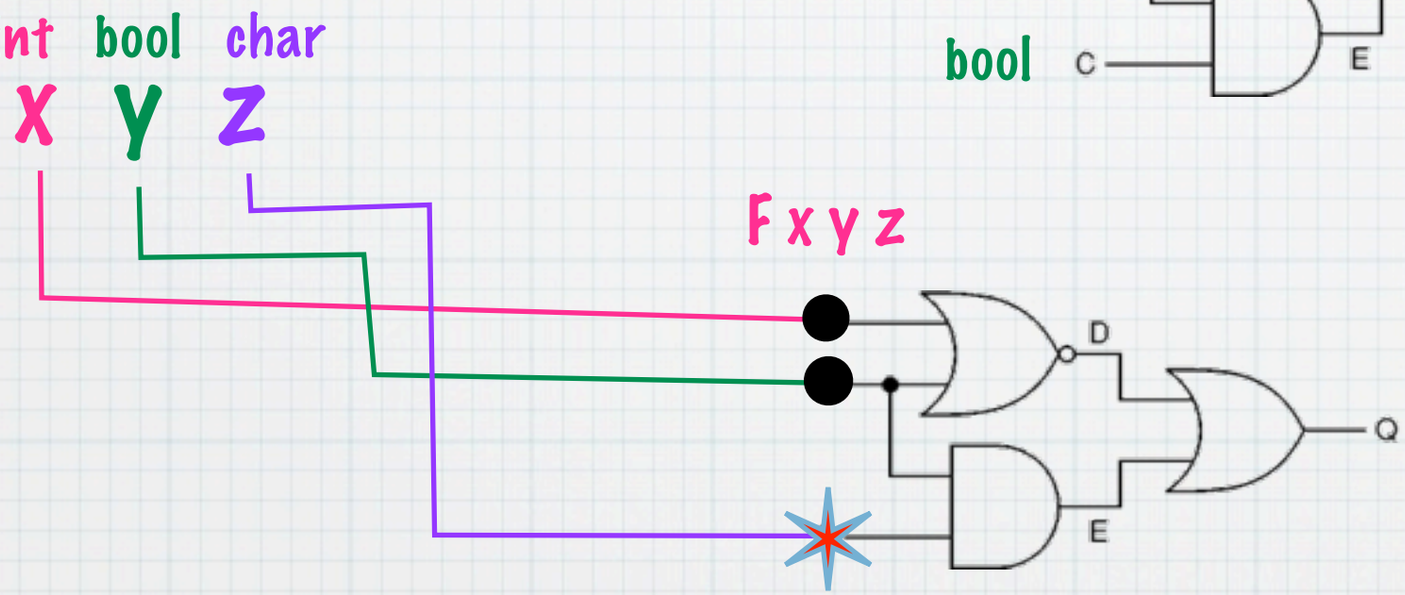

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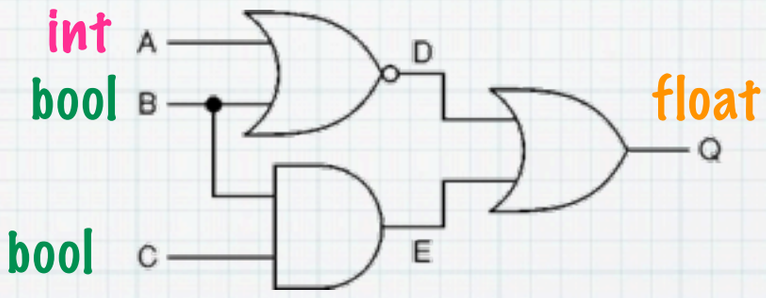

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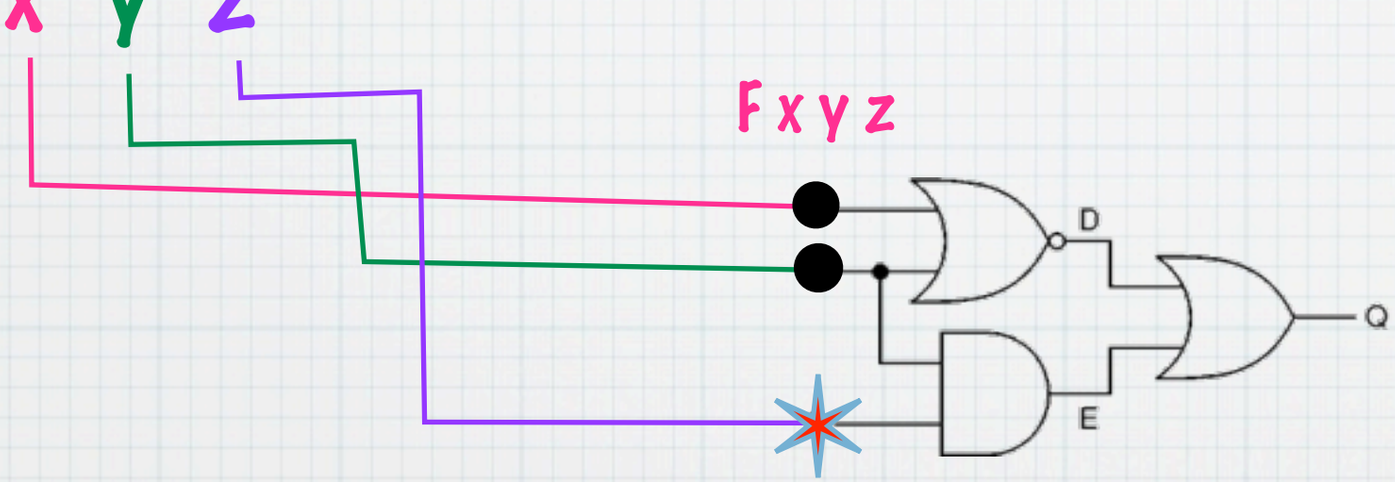

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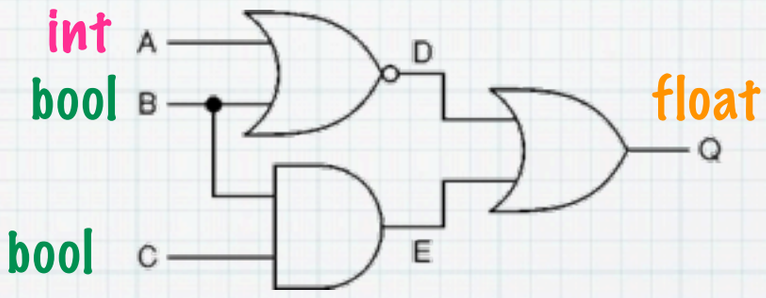

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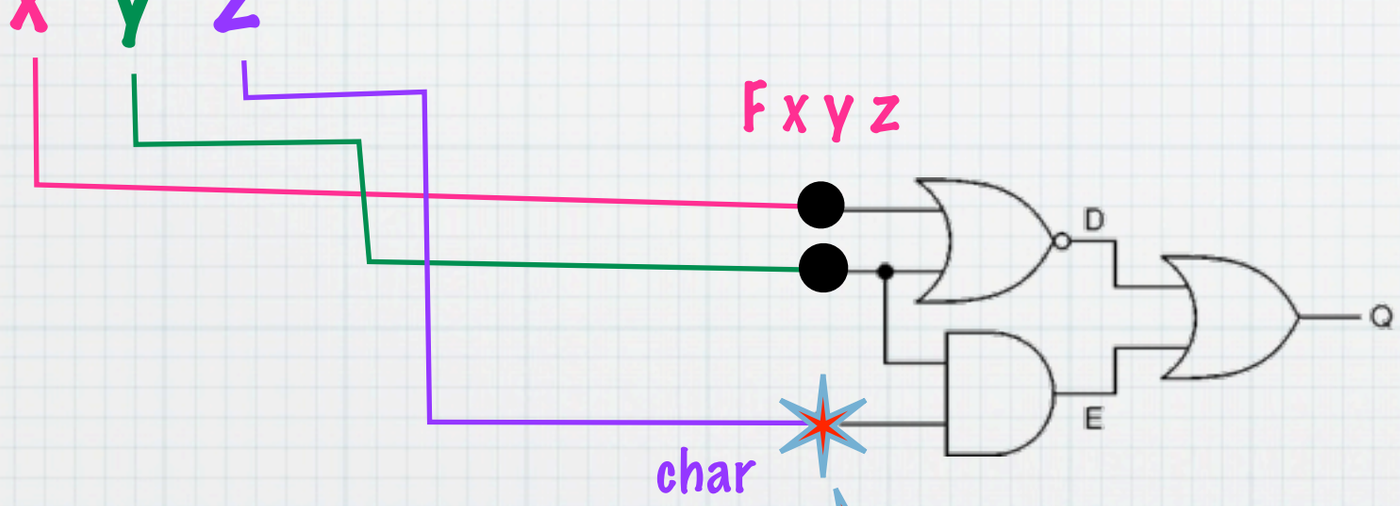

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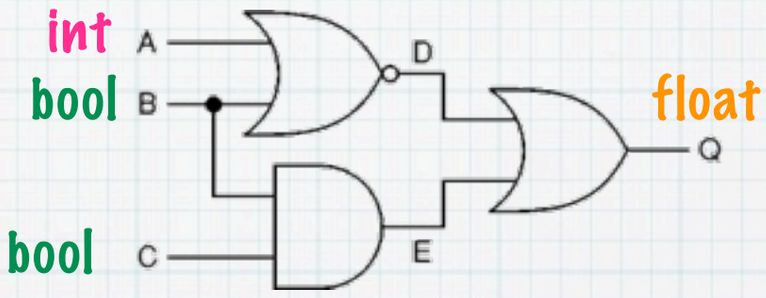

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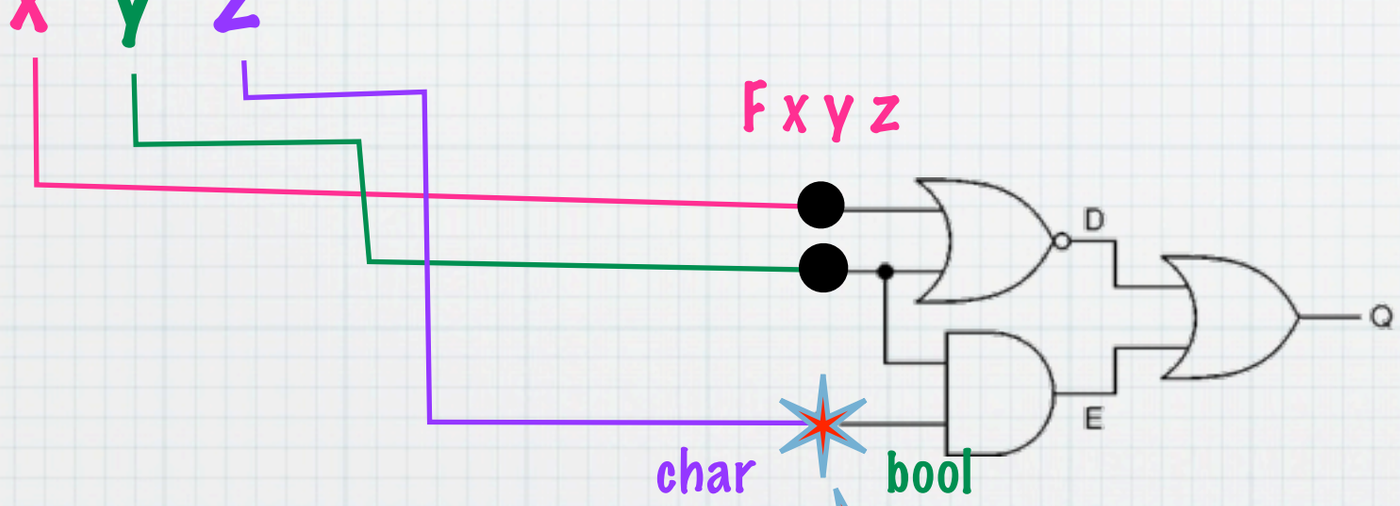

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What is Type Inference?

Given circuit with **partially labeled** pins,
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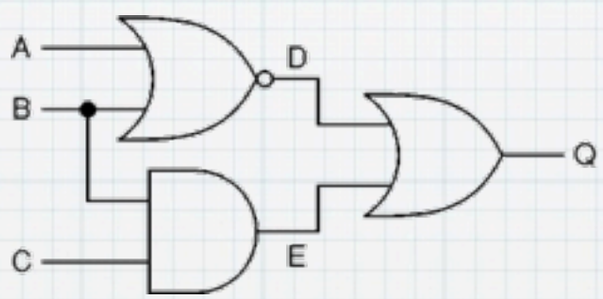

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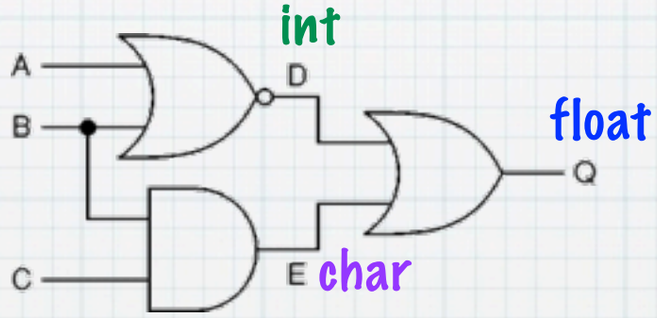

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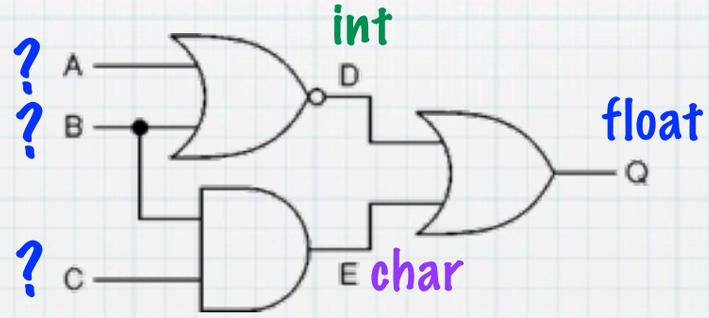

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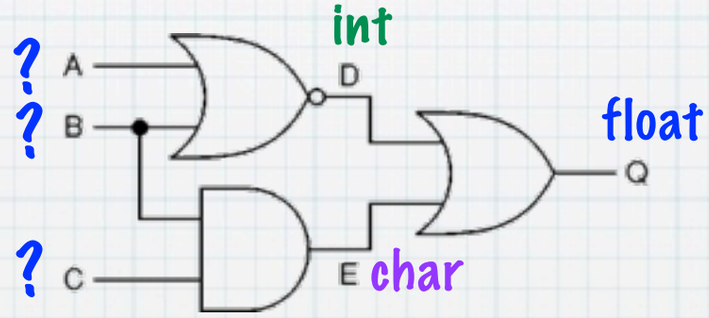

In essence a logic programming problem

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* Example
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a function from a to b

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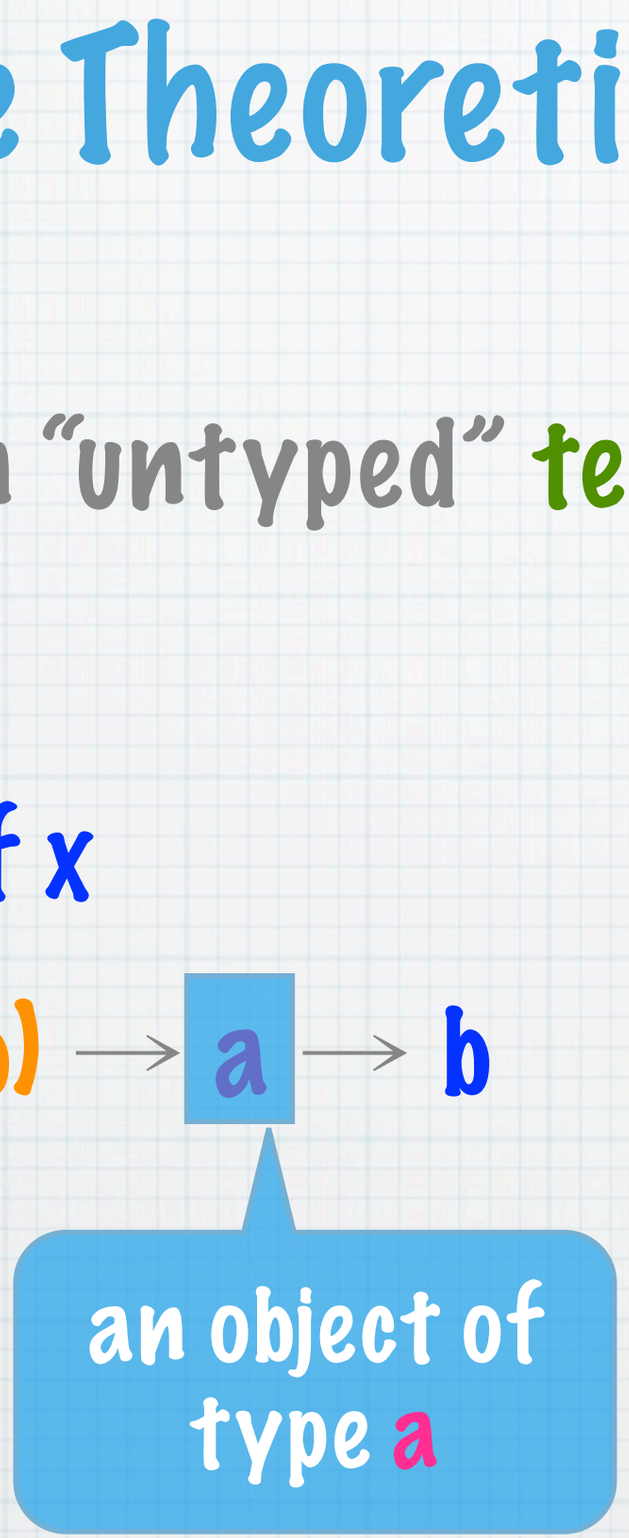

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returns an object of type b

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a and b are called "type variables" (white lie)

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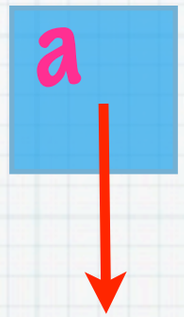

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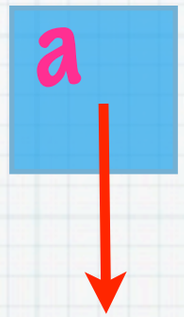

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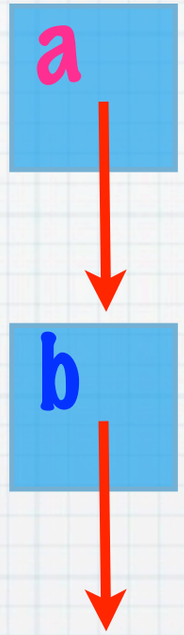

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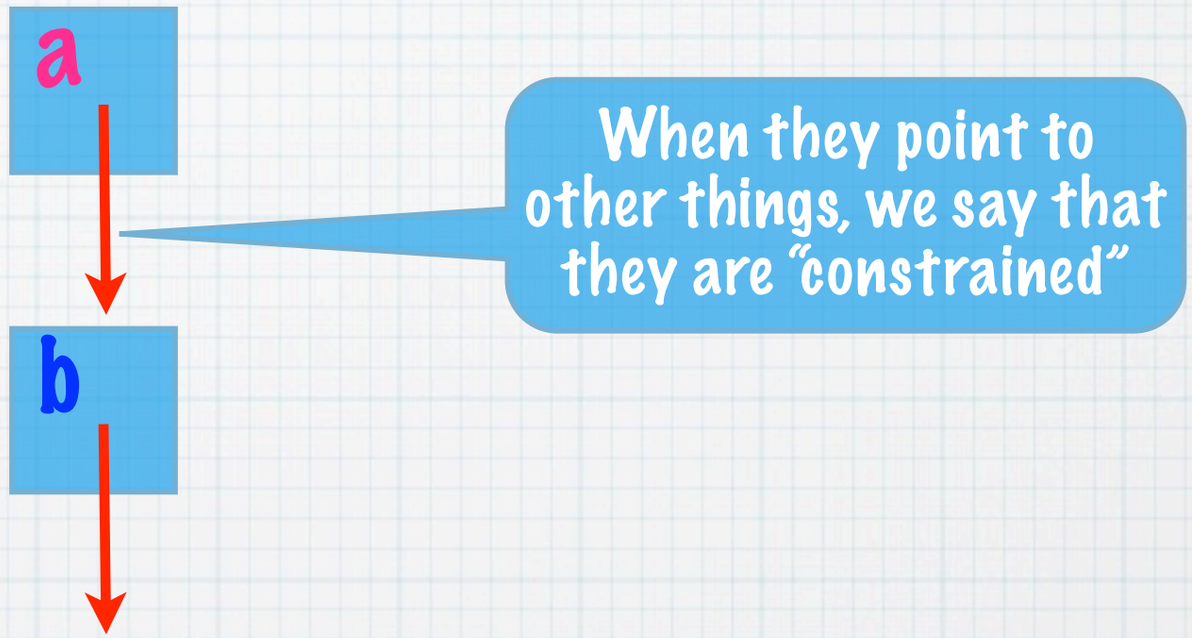

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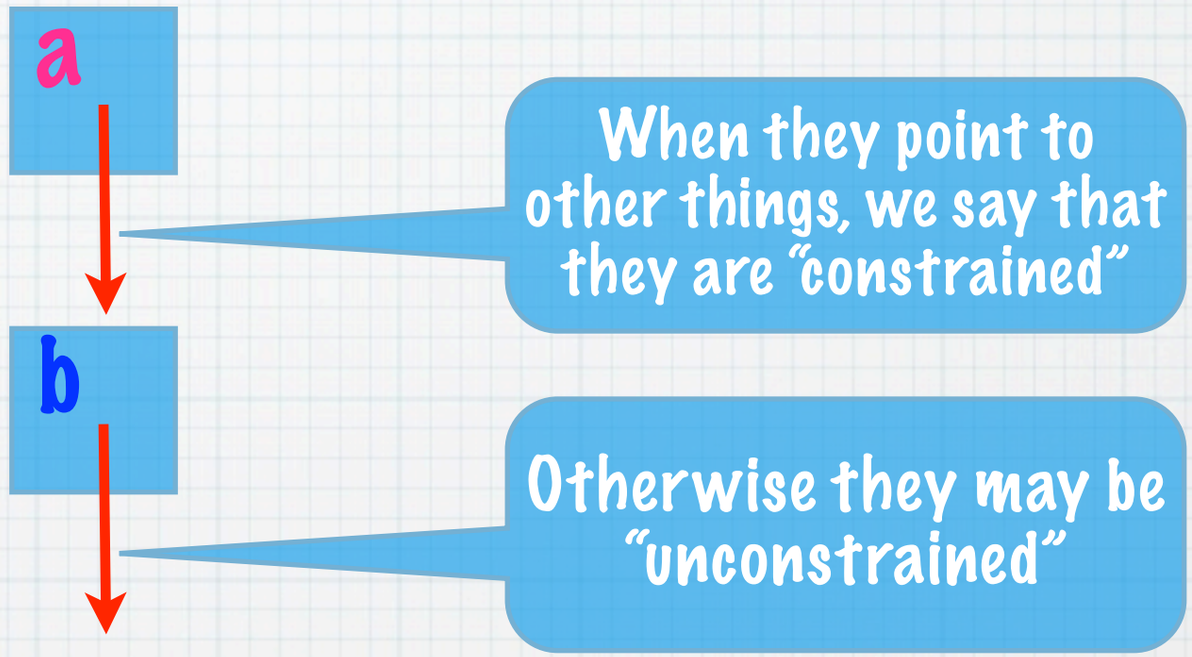

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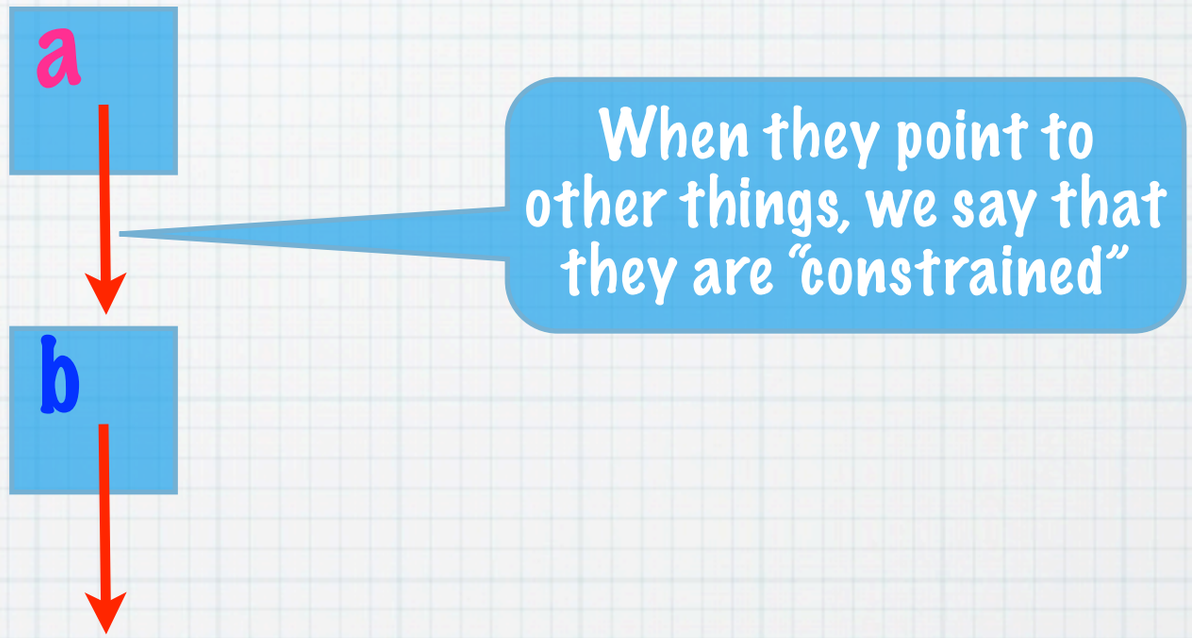

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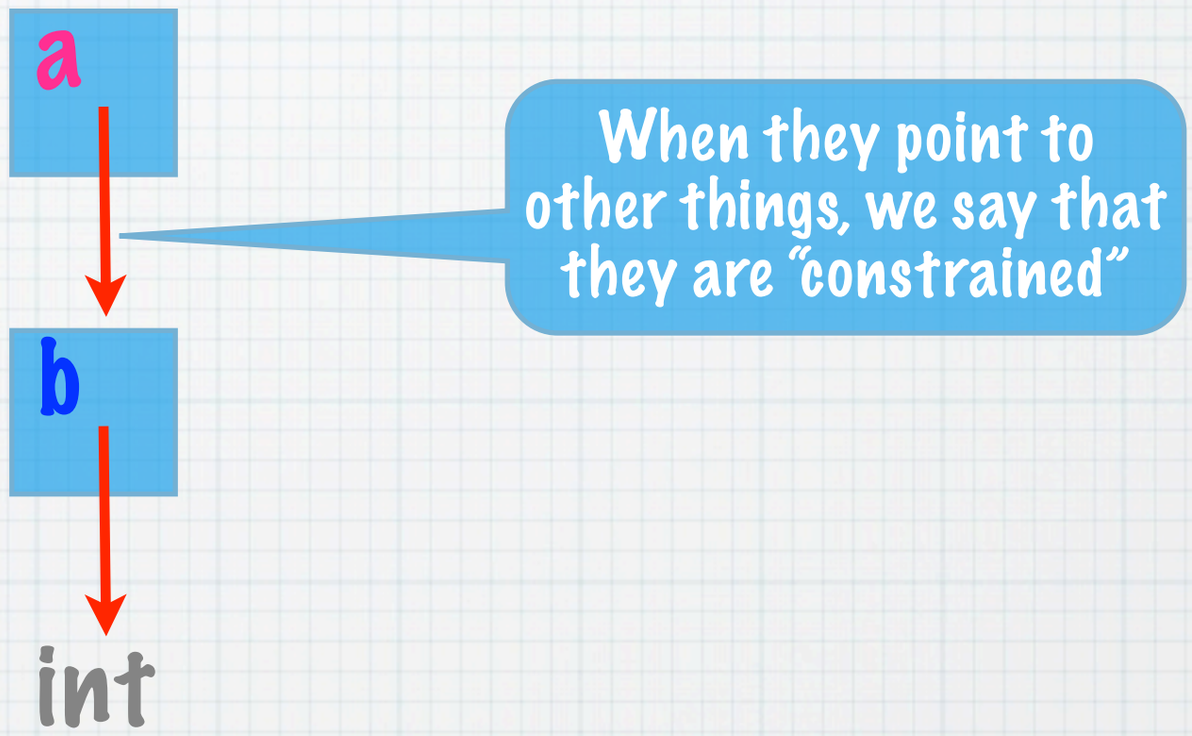

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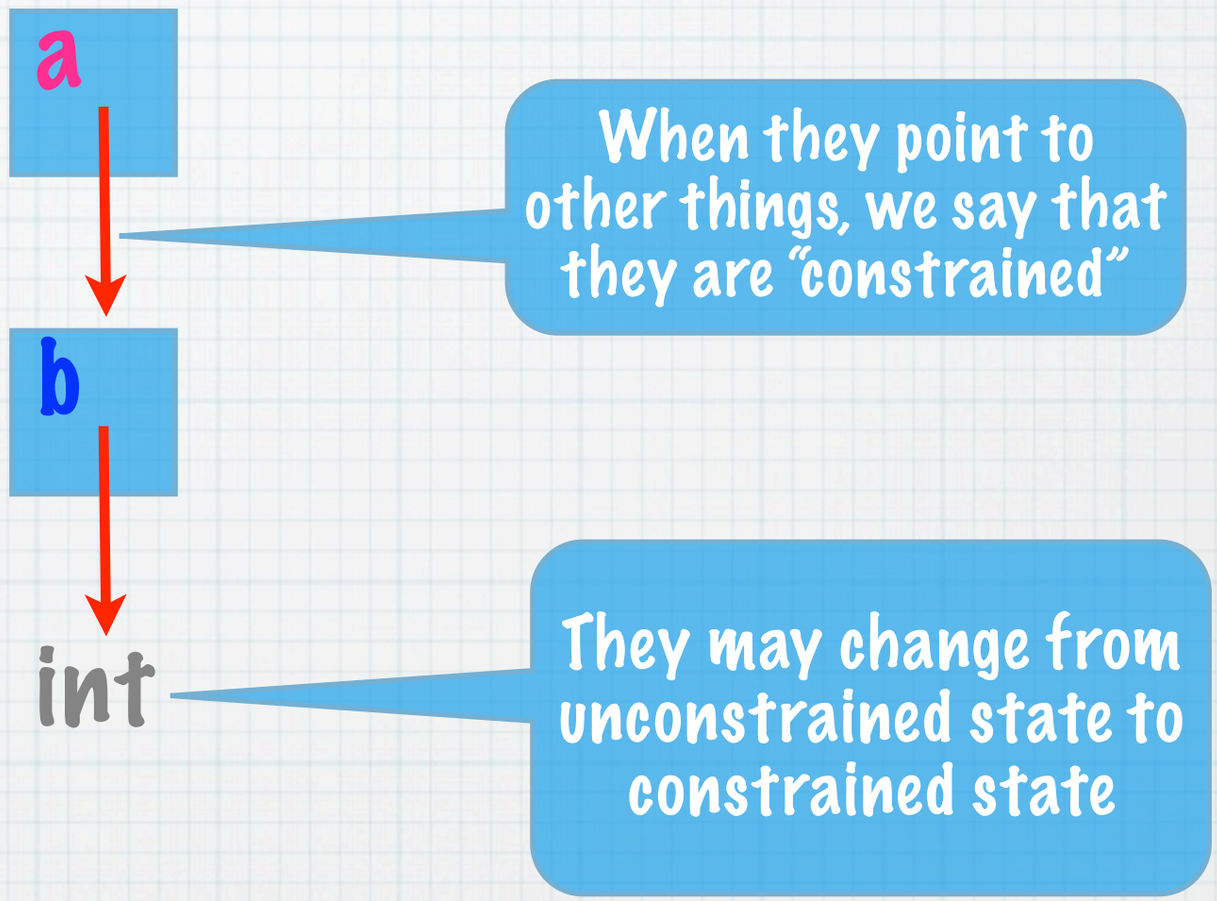

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Process the term "recursively", building a **data structure** (using pointers) from it

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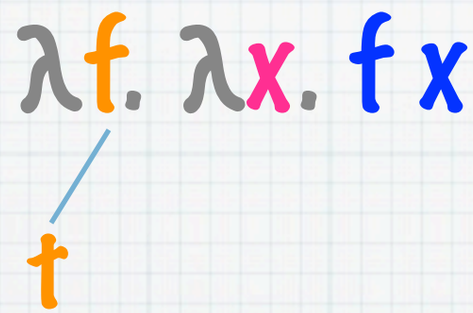

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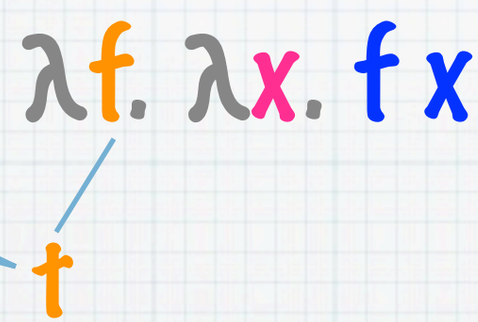
Process the term "recursively", building a data structure (using pointers) from it

Associate with each parameter a "type variable" when going down



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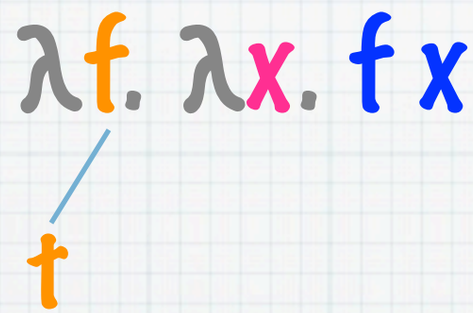

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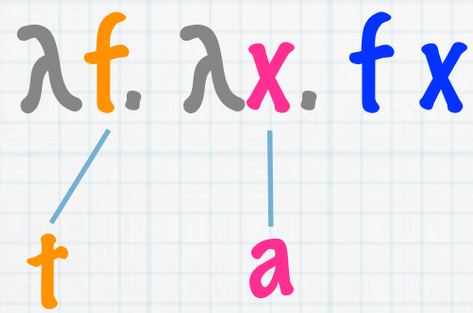

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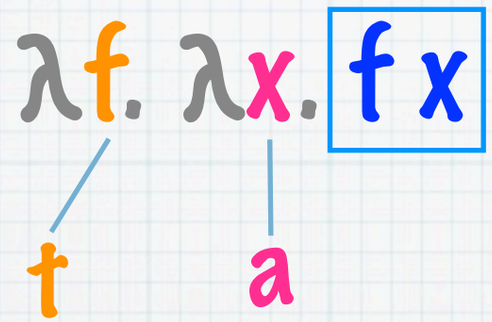

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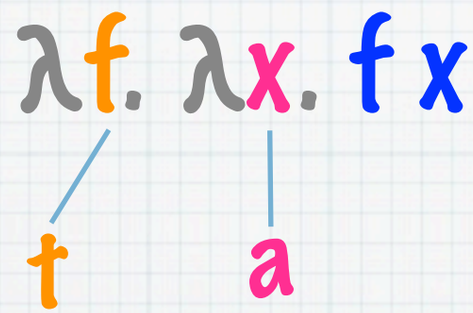

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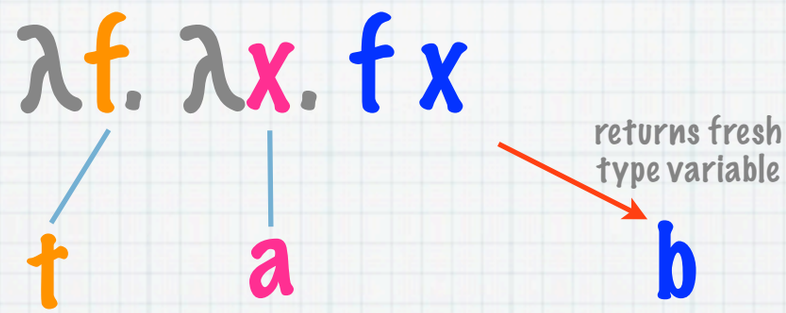

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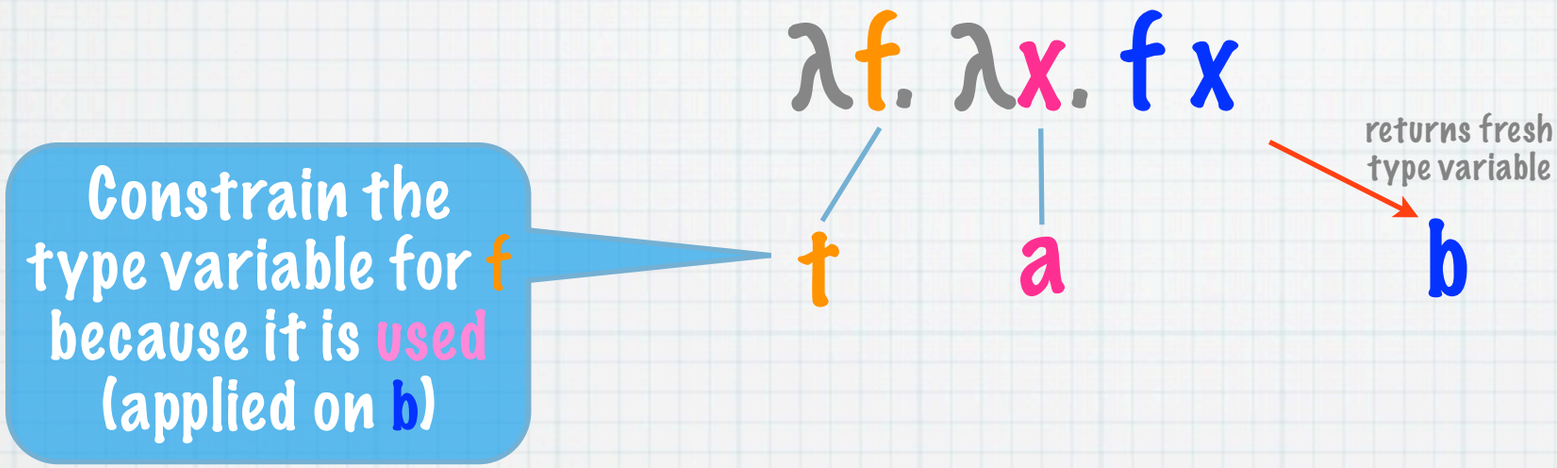

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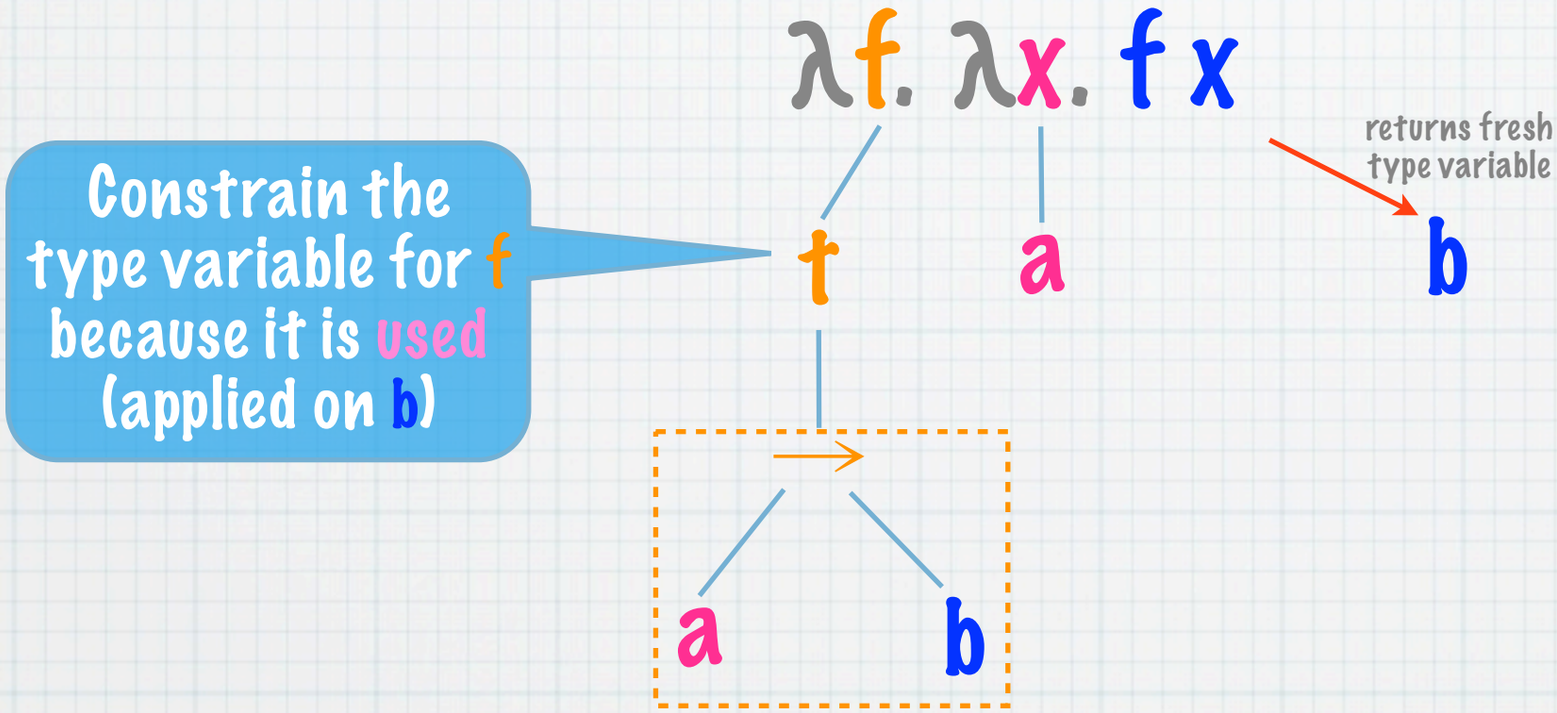

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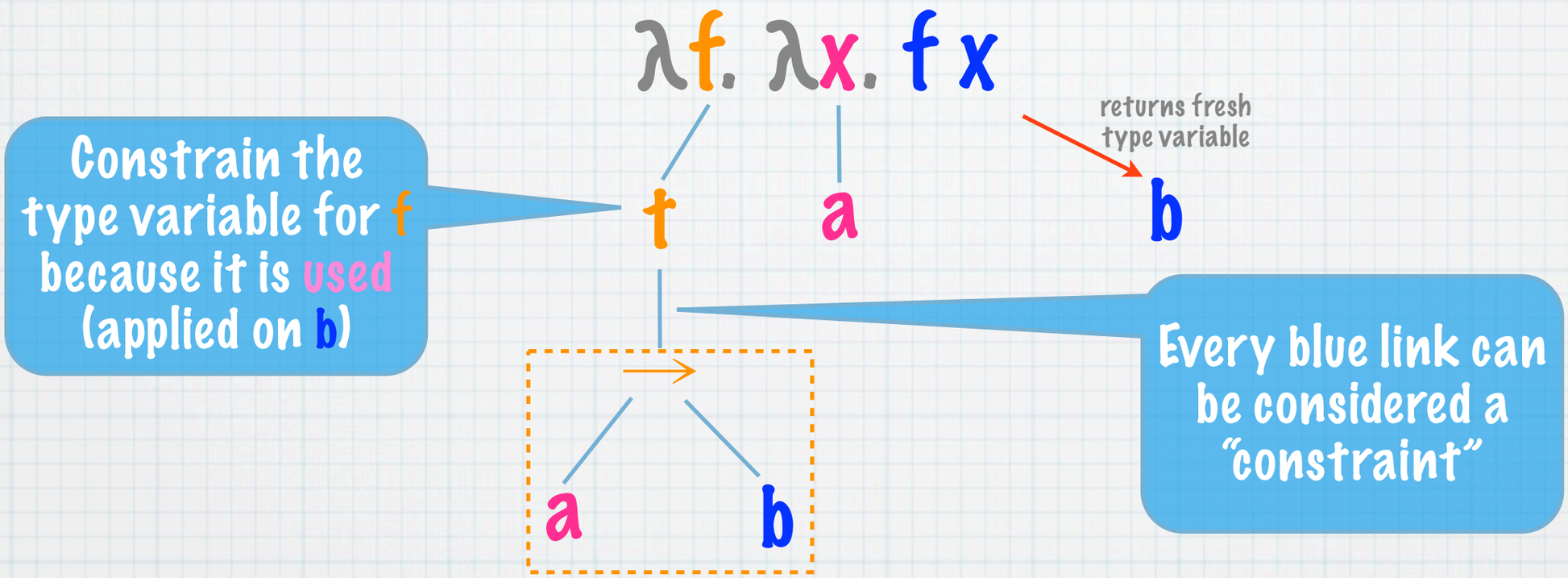

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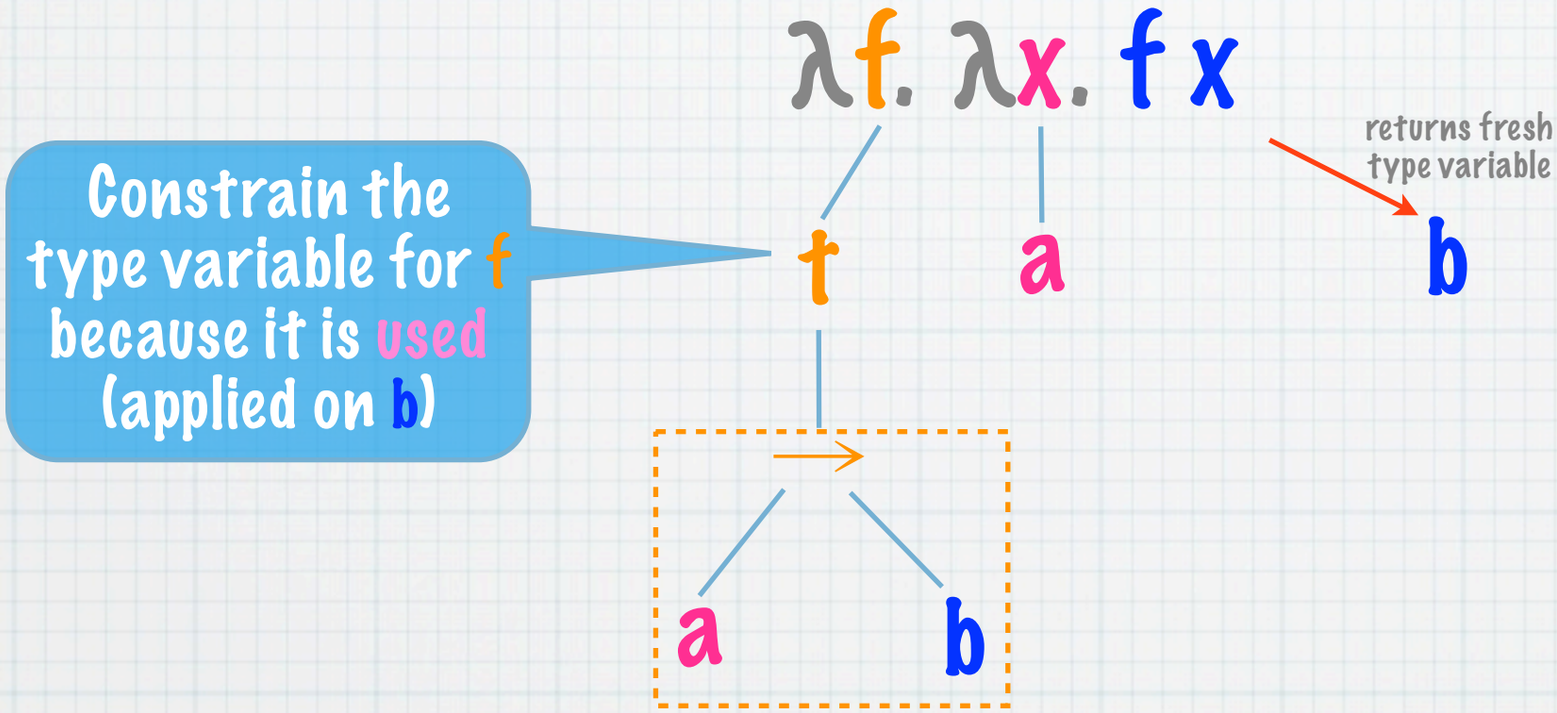

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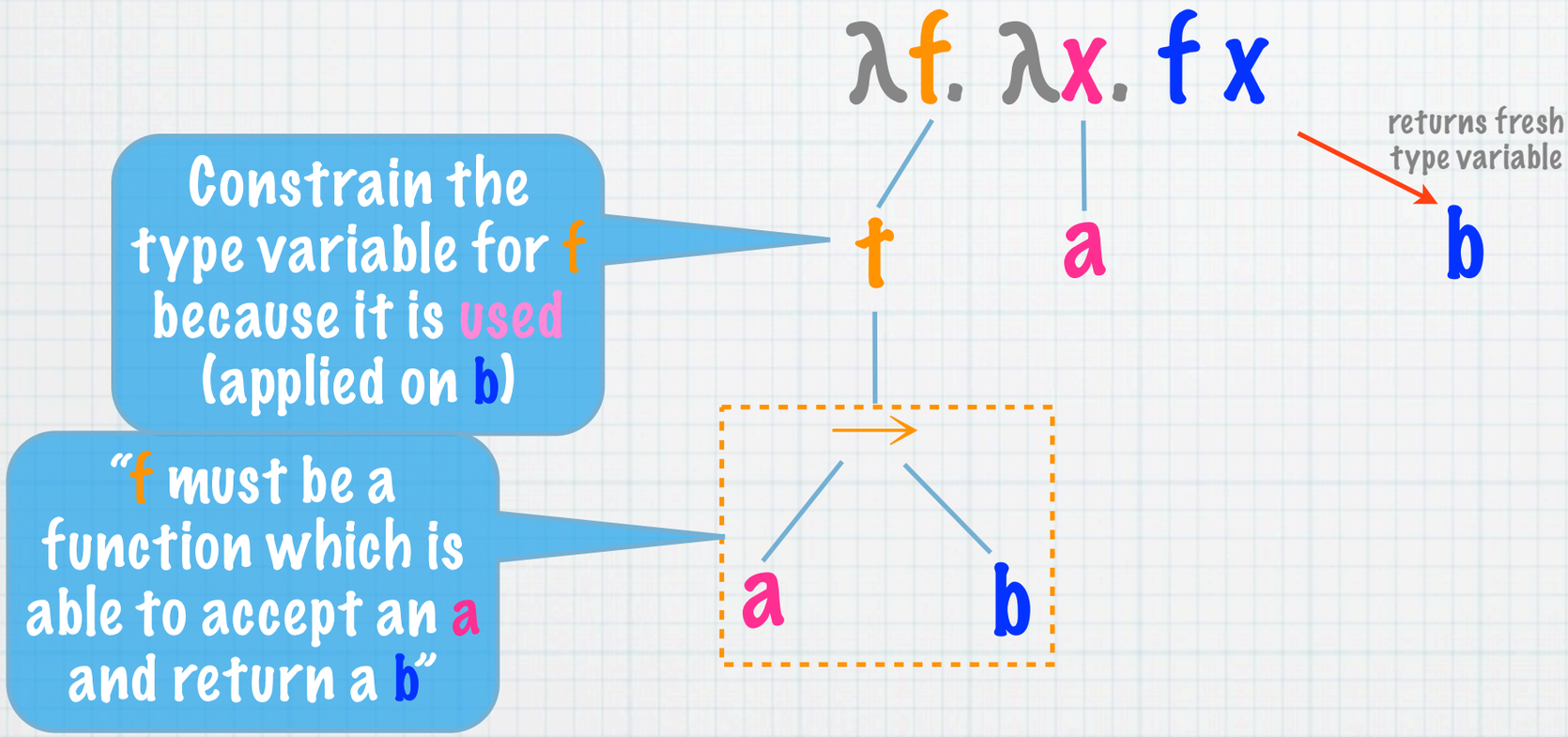

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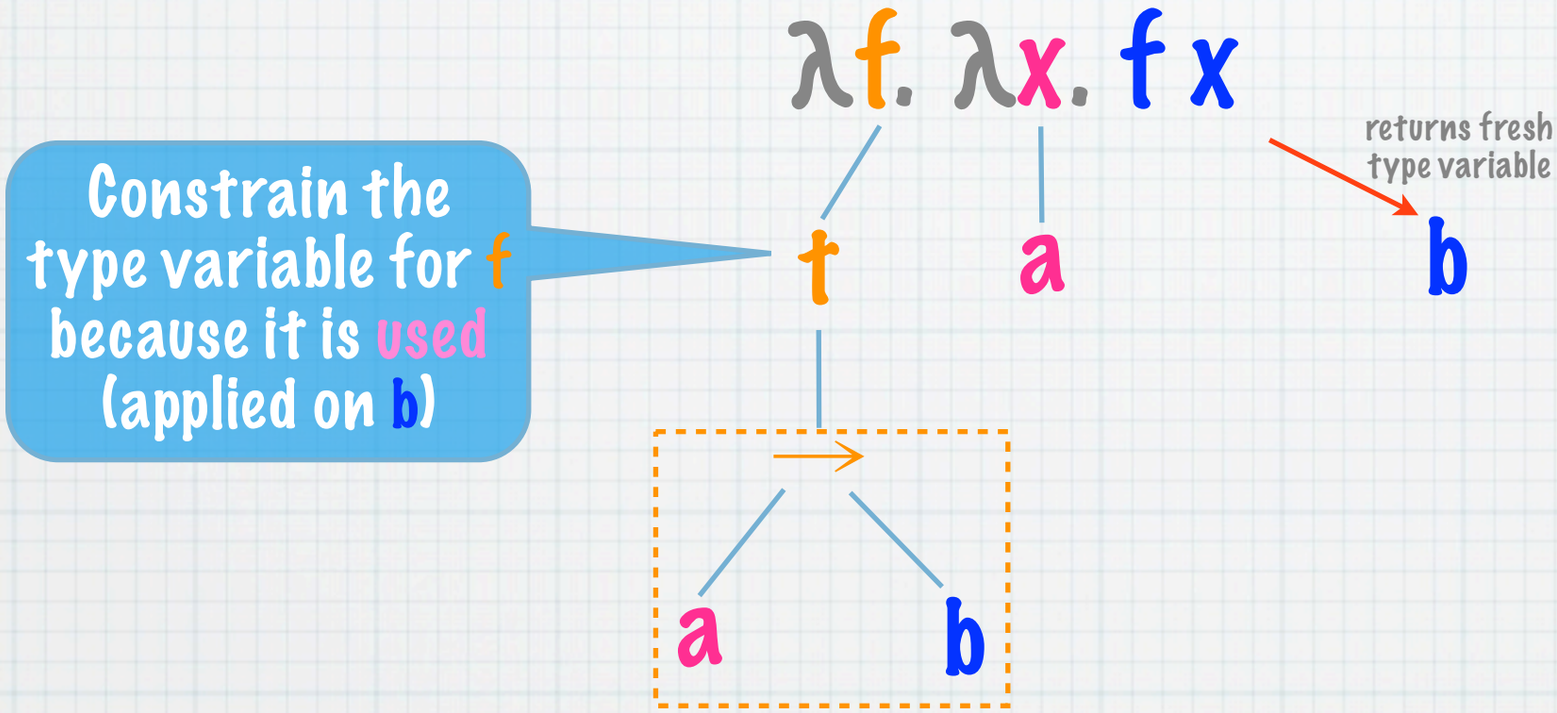

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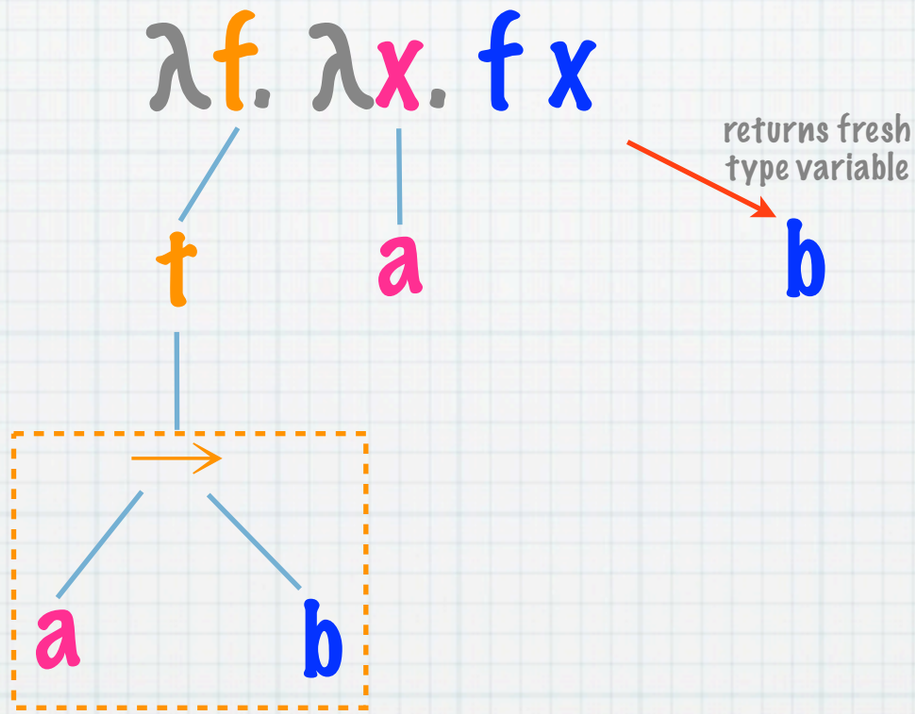

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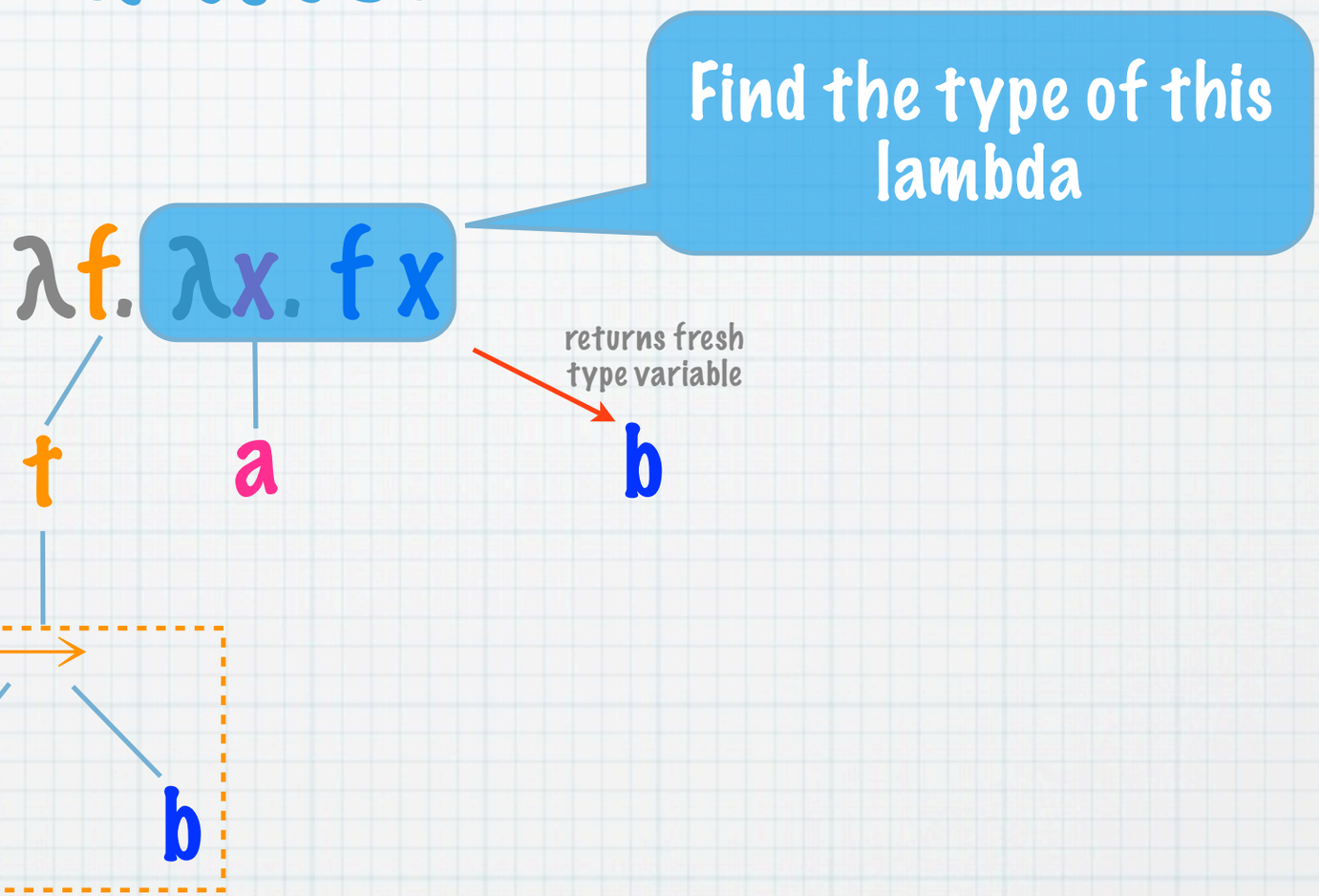
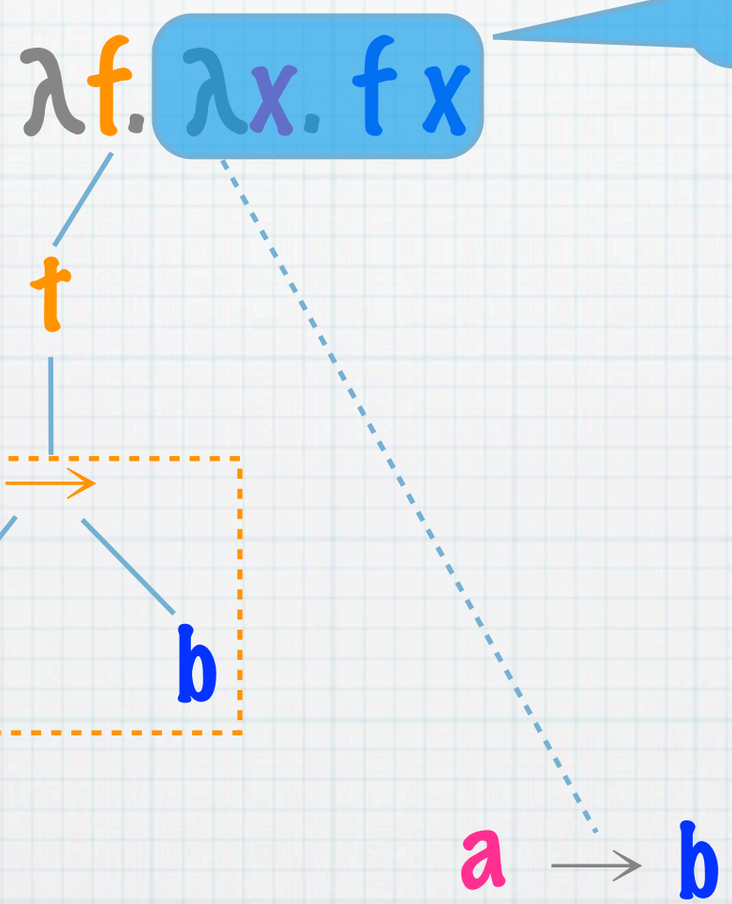


Hindley-Milner System (HMS)

Find the type of this lambda

λf. λx. f x



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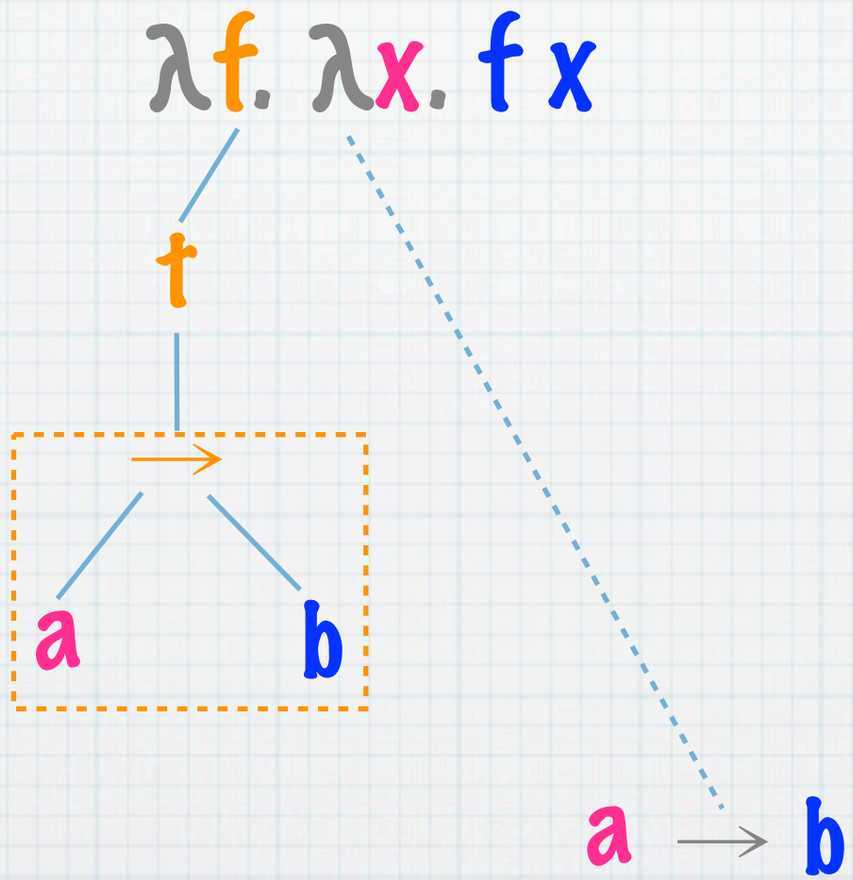

Hindley-Milner System (HMS)

Find the type of this lambda

$$\lambda f.\ \lambda x.\ f\ x$$

$$(a \to b) \to a \to b$$

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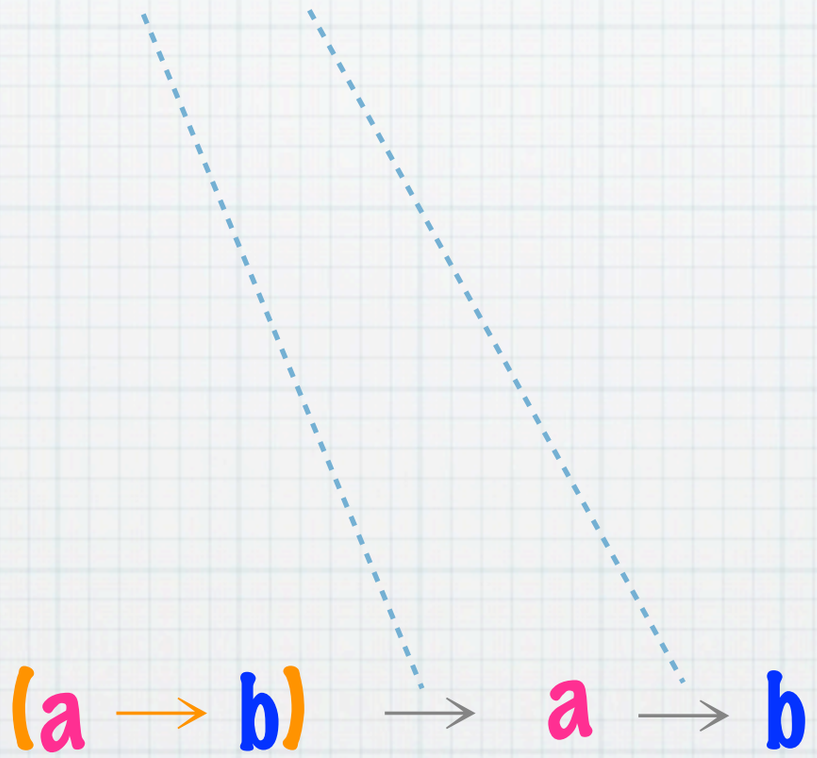

Hindley-Milner System (HMS)

$$\lambda f.\ \lambda x.\ f\ x$$

$$(a \to b) \to a \to b$$

How many kinds of arrows are there?

Hindley-Milner System (HMS)

$$\lambda f.\ \lambda x.\ f\ x$$

$$(a \to b) \to a \to b$$

Are a and b constrained?

Hindley-Milner System (HMS)

$$\lambda f.\ \lambda x.\ f\ x$$

$$(a \to b) \to a \to b$$

Can a and b be constrained?

Why do we need polymorphism?

```
let f =λx.x in
(f 1, f True)
```

Why do we need polymorphism?

If we don't have polymorphism ...

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a → a

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$a \rightarrow a$

What is f's type after the application (f 1)?

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What is f's type after the application (f 1)?

int → int

a → a

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a → a

int → int

Why?

Because a was constrained to int when we apply f to 1. Once constrained, type variables stay constrained forever.

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$a \rightarrow a$

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$int \rightarrow int$

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If we don't have polymorphism ...

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$a \rightarrow a$

a

int

Because a was constrained to int when we apply f to 1. Once constrained, type variables stay constrained forever.

Type Error !

What is f's type after the application (f 1)?

int → int

Why?

How to implement polymorphism (the HM way)

```
let f =λx.x in
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"Generalize" when we finish typing the RHS of let bindings (let-polymorphism)

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Basically, consider every unconstrained variable on the RHS as "universally quantified" (white lie)

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$\forall a.a \rightarrow a$

Each application gets a "fresh instance" of $a \rightarrow a$

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still $\forall a.\ a \to a$

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Because type variables can be constrained only in their own instances

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Is $\forall a.\ a \rightarrow a$ in fact a function in disguise?

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This is the behavior of a **function**!

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What is wrong with let-polymorphism?

For any value x of type A

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- The type [A] is for this check

What we get from let-polymorphism

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dynamic scoping

If anything feels wrong, it will go wrong ...

```
let r = ref (λx.x) in
    (r := λx.x+1; (!r)true)
```

If anything feels wrong, it will go wrong ...

What is the type of this?

```
let r = ref (λx.x) in
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```

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```
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∀a.(ref (a->a))

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If anything feels wrong, it will go wrong ...

Remember what the quantifier ∀a mean?

∀a.(ref (a->a))

```
let r = ref (λx.x) in
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```

If anything feels wrong, it will go wrong ...

Remember what the quantifier ∀a mean?

∀a.(ref (a->a))

It means make an **instance** of its body at every use

```
let r = ref (λx.x) in
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∀a.(ref (a->a))

let r = ref (λx.x) in
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ref (a_1->a_1)

If anything feels wrong, it will go wrong ...

$\forall a.(\text{ref } (a \to a))$

$\text{let } r = \text{ref } (\lambda x.x) \text{ in}$

$(r := \lambda x.x+1;\ (!r)\text{true})$

$\text{ref } (a_1 \to a_1)$

$\text{ref } (a_2 \to a_2)$

If anything feels wrong, it will go wrong ...

∀a.(ref (a->a))

let r = ref (λx.x) in
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ref (a_1->a_1)

ref (a_2->a_2)

Passed type check

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Unsound!

ML's Solution
(value restriction)

∀a.(ref (a->a))

let r = ref (λx.x) in
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No generalization for "non-values" (**ref** is a case of non-value)

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No longer make instances

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Type changes to int->int because **a** was constrained to **int**

Fails type check because int->int cannot be applied to **bool**

But ... This solves only one symptom of the problem

* There are more of them:
  * Interference with **first-class continuations**
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  * And so on ...

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Can we get rid of the root of the problem?

Mental Burden on the Programmer

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Can we do better?

The Real Problem

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ref (∀a.(a->a))

Can we type it as ref (∀a.a->a) ?

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* The **caller** should not constrain the parameter type, only the **function definition** can
* So the door to constraining the type of parameters should be "closed" at **function boundaries**
* This means: generalize should happen at **λ**, and not **LET**

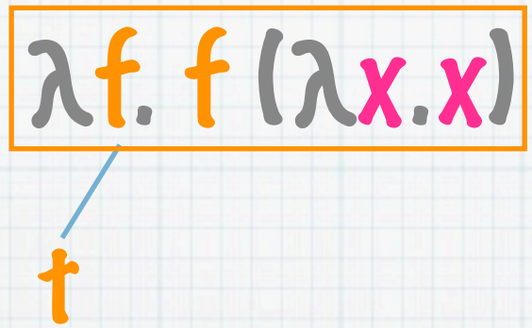
How does Genλ happen?

λf. f (λx.x)

How does Genλ happen?

$$\lambda f.\ f\ (\lambda x.x)$$

How does Genλ happen?



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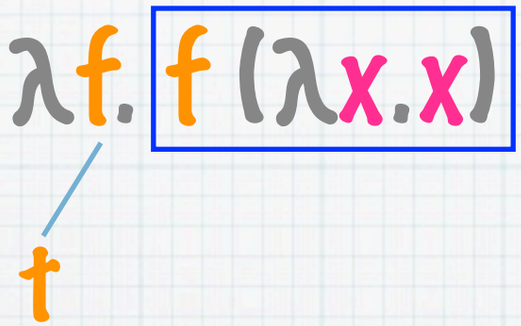

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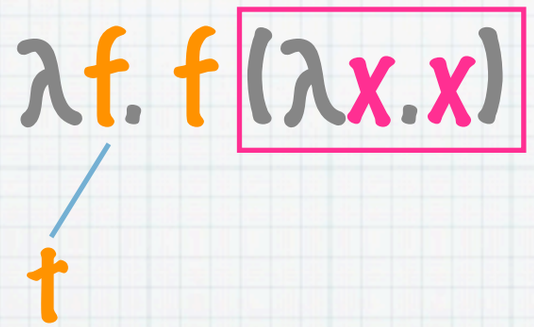

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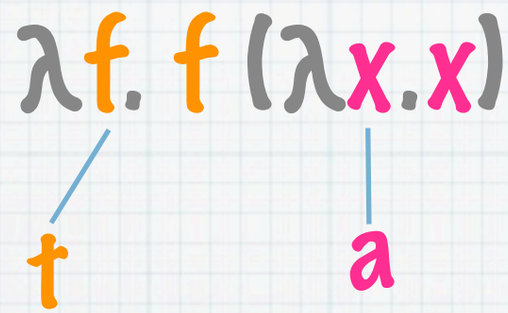

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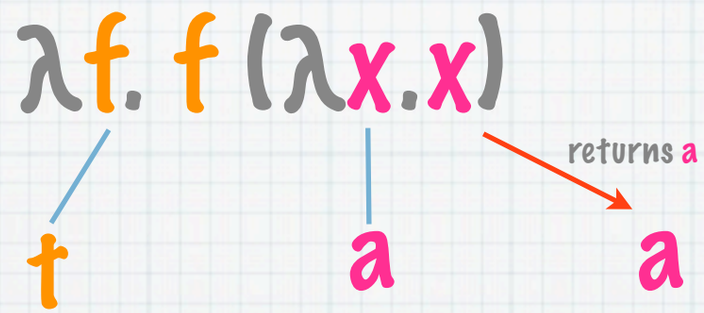

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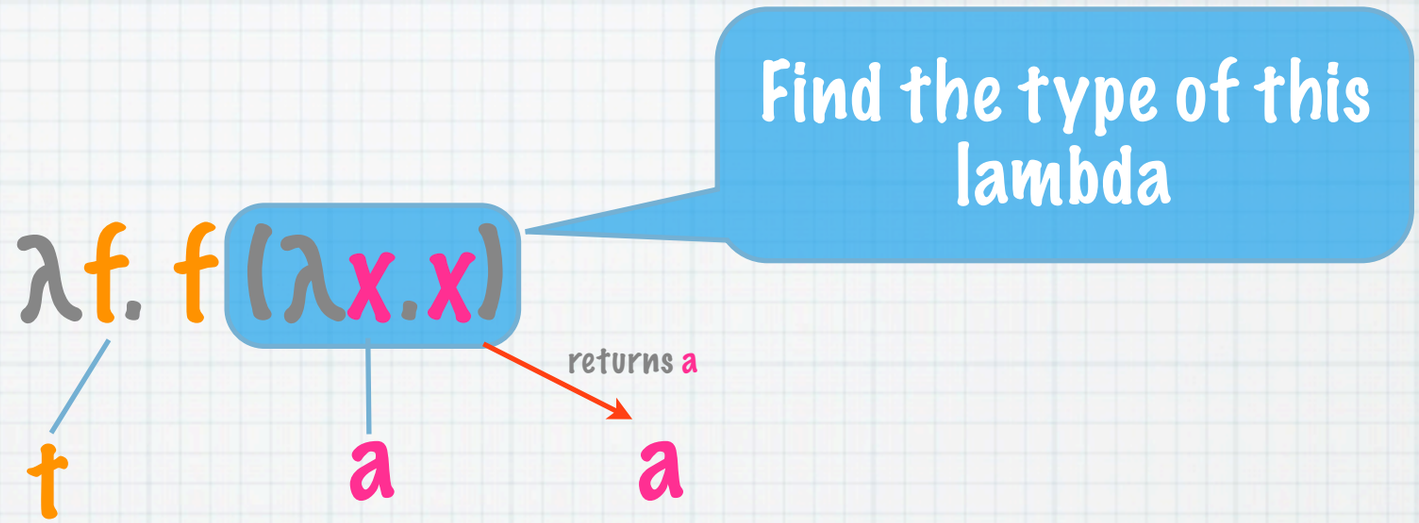


How does Genλ happen?

Find the type of this lambda

λf. f (λx.x)

f

a → a

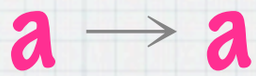
How does Gen happen?

Backing out of the scope of x

Find the type of this lambda

$$\lambda f.\ f\ (\lambda x.x)$$

$$f$$



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Backing out of the scope of x

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t

a can no longer be constrained because x is going out of scope and no outer references to a exists, so generalization happens

$a \rightarrow a$

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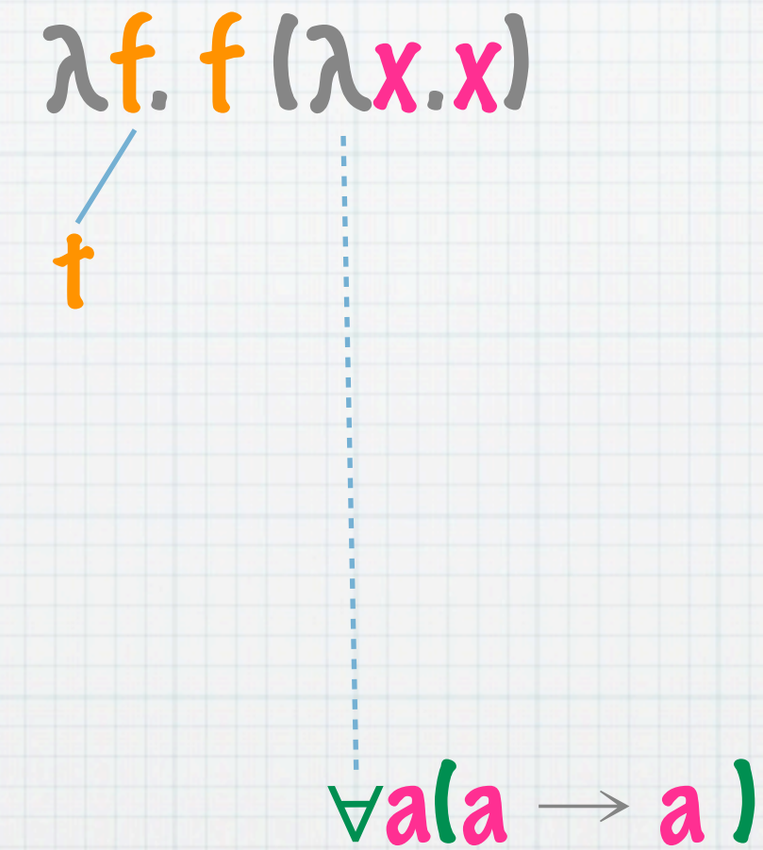

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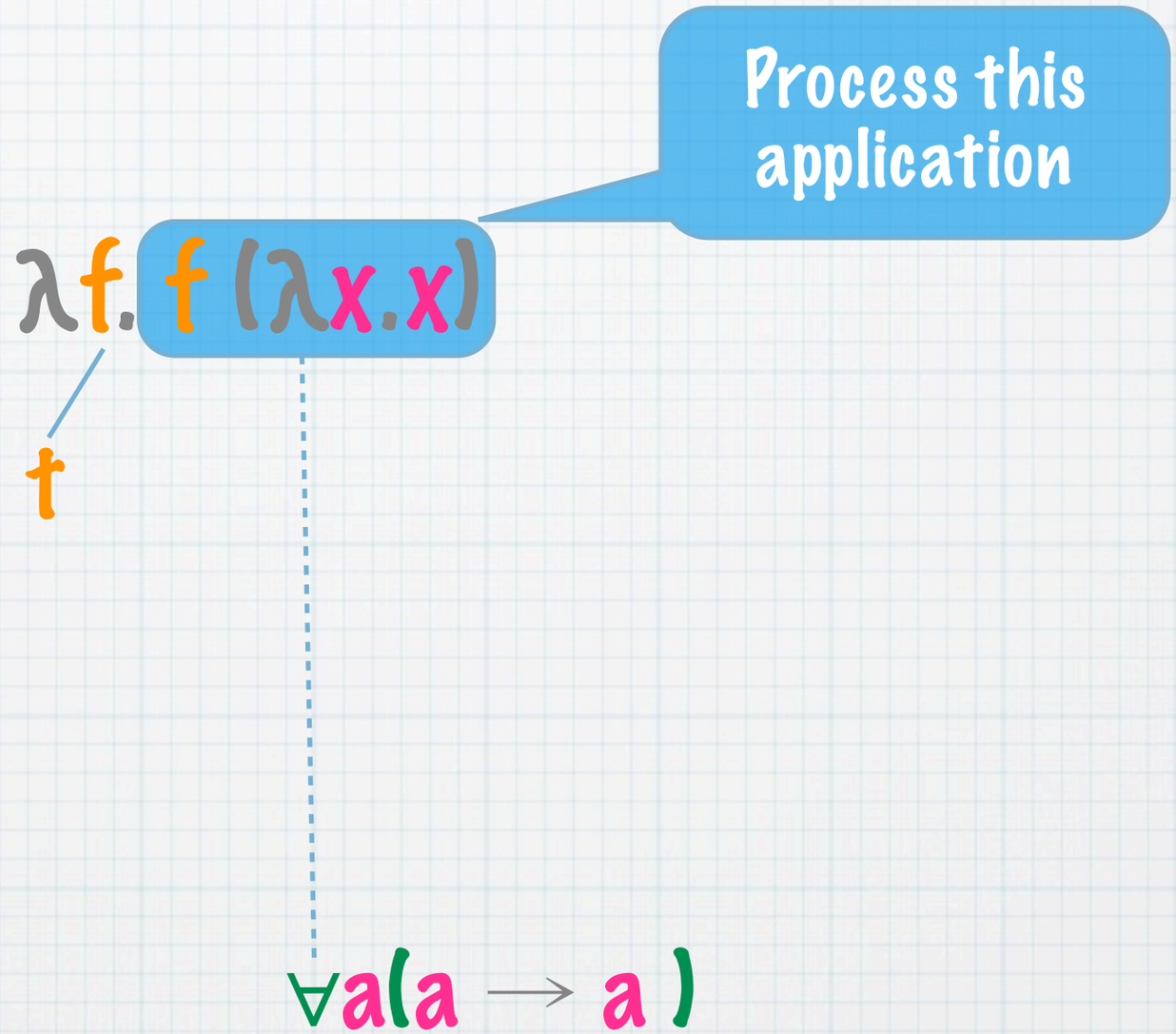

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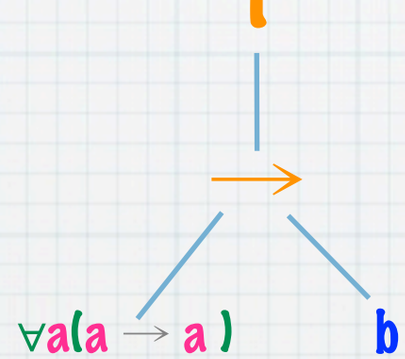
Process this application

λf. f (λx.x)

t



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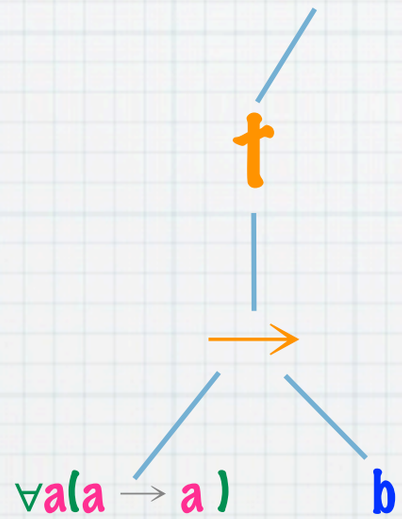

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$$((\forall a.a \to a) \to b) \to b$$

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$$\forall b.((((\forall a.a \to a) \to b) \to b) \to b)$$

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λf. f (λx.x)

∀b.(((∀a.a → a) → b) → b)

This is a "higher ranked" type

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The Difference of Two Types

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★ Gen$_\lambda$ type contains more information

★ We can know from the type that the argument will be applied to a polymorphic function

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★ Gen$_\lambda$ type contains **more information**

★ We can know **from the type** that the argument **will be** applied to a **polymorphic function**

Is "more information" always good?

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An example of such a system is intersection types

What are Intersection Types?

λf. (f 1, f true)

Is typed as: (→ a ∧ → b) → (a, b)

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supposed to produce (1, true)

What are Intersection Types?

$\lambda f.\,(f\ 1,\ f\ \text{true})$ "f is simultaneously $\text{int} \to a$ and $\text{bool} \to b$"

Is typed as: $(\ \text{int} \to a \wedge \text{bool} \to b) \to (a, b)$

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The problem is, how to get (int, bool)
given $(\ \text{int} \to a \wedge \text{bool} \to b) \to (a, b)$
and $\forall a.a \to a$

What are Intersection Types?

Answer: "apply" $\forall a.a \to a$ twice,
once to **int**, the other time to **bool**

When applied on $\lambda x.x$: $(\lambda f.\ (f\ 1,\ f\ true))\ (\lambda x.x)$

The problem is, how to get (**int**, **bool**)
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The Rest of the Story

* I have only shown intersection types which doesn't capture the **control flow** information
* We can capture control flow information in the type, removing **idempotence**
* The resulting systems will be even more powerful, able to type self applications like $\lambda x.xx$ or $\lambda x.xxx$
* To be continued ...